Indian Statistical Institute, Bangalore Centre
M.Math I Year, First Semester
Mid-Sem Examination
Algebra I

Time: 3 Hours   September 26, 2011   Instructor: N.S.N.Sastry

Total Mark is : 100

Note: Answer all questions. Your answers should be complete and clearly written.

1. Define a t-transitive permutation group. Give an example of a 2-transitive group which is not 3-transitive.Justify.   [5+8]

2. Let G be a group of order 21.

   (a) Show that it contains a normal subgroup of order 7.

   (b) Determine the automorphism group of a group of order 7.

   (c) Determine all finite groups of order 21.   [4+4+9]

3. Let R be the ring of all 2×2 - matrices with entries from a field F.

   (a) Show that it has no 2-sided proper ideals.

   (b) Show that its only proper left ideals are:

$$
\begin{aligned}
I_1 &= \{(a_{ij}) \in R \ : \ a_{i1} = 0 \text{ for } i = 1, 2\} \\
\text{and } I_2 &= \{(a_{ij}) \in R \ : \ a_{j2} = 0 \text{ for } j = 1, 2\}
\end{aligned}
$$

          [10+10]

4. Construct a subring R of the field $\mathbb{Q}$ of rational numbers in which 2 is invertible but not 3.   [15]

5. (a) Find a polynomial of degree 3 with coefficients in the field $\mathbb{F}_2$ of order 2 which is irreducible over $\mathbb{F}_2$.

   (b) Using the polynomial in (a), write down the multiplication table for a field of order 8.   [8+12]

6. Let H be a subgroup of a finite group G. Assume that the index of H in G is the smallest prime division of the order of G. Show that H is a normal subgroup of G.   [15]

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